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Non-Abelian Brane Worlds: The Open String Story

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Abstract. *We extend the string model building rules for the construction of chiral supersymmetric Type I compactifications on smooth Calabi-Yau manifolds. These models contain stacks of D9-branes endowed with general stable $U(n)$ bundles on their world-volume and D5-branes wrapping holomorphic curves on the Calabi-Yau.*

Key words: *String compactifications, D-branes*

1. INTRODUCTION

Considerable effort has gone into the stringy construction of semi-realistic intersecting D-brane models during the last five years (see [1, 2, 3, 4] for reviews and further references.). Though people have tried hard, it is fair to say that so far no completely satisfactory model has emerged. However, before jumping to conclusions, one should keep in mind that all the effort has essentially focused on a very tiny subset of concrete models. For simplicity one has studied in detail the restricted class of a handful of toroidal orbifold spaces with a certain subset of supersymmetric D-branes. In the T-dual (mirror symmetric) picture these intersecting D-brane models are Type IIB orientifolds with magnetized D9-branes. The magnetized D-branes carry $U(1)$ bundles on their world-volume, and the supersymmetry condition in flat space is just the abelian MMMS equation [5].

Heterotic string model building and its recent successes [6, 7] demonstrate that more generally it might prove fruitful to consider arbitrary Calabi-Yau spaces equipped with vector bundles with structure groups of higher rank. In fact, we have studied compactifications of both the $E_8 \times E_8$ [8](see also [9, 10]) and the $SO(32)$ [11] heterotic string endowed with general $U(n)$ bundles. Note that before, mostly the $E_8 \times E_8$ heterotic string with $SU(n)$ bundles was considered. Of course, one expects that the general results for the $SO(32)$ heterotic string [11] can be translated via S-duality to compactifications of the Type I string. In this article we explicitly verify this statement and derive the model building rules for the Type I compactifications on smooth Calabi-Yau manifolds directly from the Type IIB and D-brane

perspective. Here we focus just on usual Ω orientifolds, but the result for more general orientifolds including an Ω-dressing by some holomorphic involution of the Calabi-Yau can be derived in a similar fashion.

The aim of this article is to make clear that the class of intersecting/magnetized D-brane models constitutes only a tiny subset of the more general class of string vacua obtained by compactifying general Type IIB orientifolds on smooth Calabi-Yau spaces and introducing D-branes endowed with general vector bundles with unitary structure groups. Concretely, we summarize the model building rules for the construction of such models. In particular, in Section 2, by dimensional reduction of the Chern-Simons terms on D-branes, we derive the 10-form and 6-form tadpole cancellation conditions. In Section 3 we provide rules for computing the chiral as well as non-chiral massless open string spectrum, which due to the Riemann-Roch-Hirzebruch theorem is determined by the Euler characteristics of various vector bundles. In addition to the D9-branes, we also allow for D5-branes wrapping effective 2-cycles on the Calabi-Yau. These branes carry symplectic gauge groups, which allows us in Section 4 to compute the K-theory constraints from the vanishing of the global Witten anomaly on these D5-branes. In Section 5 we summarize the main formulas for the cancellation of the various abelian anomalies. These are used to derive in Section 6 the perturbative expressions for the Fayet-Iliopolous (FI) terms and in Section 7 the holomorphic gauge kinetic functions. These two quantities are given by the same expression as appearing in the Π-stability condition for B-type branes [12]. In Section 8 we summarize the supersymmetry conditions for the magnetized D9-branes and conclude in Section 9 with some outlook on the next steps to concrete semi-realistic string model building using the set-up.

2. TADPOLE CANCELLATION

We consider compactifications of the Type I string to four space-time dimensions on a Calabi-Yau manifold X. We start with the ambient model, which is the Type IIB string divided by the world-sheet parity transformation $\Omega : (\sigma, \tau) \rightarrow (-\sigma, \tau)$. As is well known, this induces a tadpole for the Ramond-Ramond (R-R) 10-form, C_{10}, and, since the Calabi-Yau is generically curved, an induced tadpole for the 6-form C_6. Quantitatively, these tadpoles are given by the CS-terms on the $O9$-plane [13, 14]

$$S_{O9}^{CS} = -32\, \mu_9 \int_{\mathbb{R}^{1,3}\times X} \left(\sum_{n=0}^{2} C_{4n+2} \right) \wedge \sqrt{\hat{\mathcal{L}}\left(\frac{\mathcal{R}}{4}\right)}, \tag{1}$$

where $\mathcal{R} = -i\ell_s^2 R$ with the string length defined as $\ell_s = 2\pi\sqrt{\alpha'}$. The Hirzebruch genus $\hat{\mathcal{L}}$ is defined as

$$\sqrt{\hat{\mathcal{L}}\left(\frac{\mathcal{R}}{4}\right)} = 1 + \frac{\ell_s^4}{192\,(2\pi)^2}\,\mathrm{tr}R^2 + \frac{\ell_s^8}{73728\,(2\pi)^4}\left(\mathrm{tr}R^2\right)^2 - \frac{\ell_s^8}{92160\,(2\pi)^4}\left(\mathrm{tr}R^4\right). \quad (2)$$

The traces are taken over the fundamental representation of the Lorentz group $SO(1,9)$.

In order to cancel these tadpoles, one introduces D9-branes endowed with holomorphic vector bundles (coherent sheaves) on their world-volume. More concretely, we take stacks of $M_i = N_i\, n_i$ branes and diagonally turn on $U(n_i)$ holomorphic vector bundles V_i, breaking the observable gauge group to $\prod_i U(N_i)$. If the gauge field on such a stack is F_i, then under the action of Ω this stack is mapped to a different stack with gauge field $-F_i$. Therefore, we have to introduce these stacks in pairs with vector bundles V_i and V_i^* supported on their world-volume.

The Chern-Simons action on the D9-branes reads

$$S^{CS}_{D9_i} = 2\,\mu_9 \int_{\mathbb{R}^{1,3}\times X} \left(\sum_{n=0}^{2} C_{4n+2}\right) \wedge \mathrm{ch}(i\mathcal{F}_i) \wedge \sqrt{\hat{\mathcal{A}}\,(\mathcal{R})} \quad (3)$$

with $\mathcal{F} = -i\ell_s^2 F$, $\mu_9 = \frac{1}{(2\pi)^9\alpha'^5}$ and

$$\mathrm{ch}_k(i\mathcal{F}_i) = \frac{\ell_s^{2k}}{k!\,(2\pi)^k}\,\mathrm{Tr}_{M_i}(F_i^k), \quad (4)$$

$$\sqrt{\hat{\mathcal{A}}\,(\mathcal{R})} = 1 - \frac{\ell_s^4}{96\,(2\pi)^2}\,\mathrm{tr}R^2 + \frac{\ell_s^8}{18432\,(2\pi)^4}\left(\mathrm{tr}R^2\right)^2 + \frac{\ell_s^8}{11520\,(2\pi)^4}\left(\mathrm{tr}R^4\right). \quad (5)$$

In addition, we allow for stacks of $2N_a$ $D5$-branes wrapping holomorphic 2-cycles, Γ_a, on X. The Chern-Simons action on the D5-branes reads

$$S^{CS}_{D5_a} = -\mu_5 \int_{\mathbb{R}^{1,3}\times\Gamma_a} \left(\sum_{n=0}^{1} C_{4n+2}\right) \wedge \left(2\,N_a + \frac{\ell_s^4}{2\,(2\pi)^2}\mathrm{Tr}_{SP}(F_a^2)\right) \wedge \frac{\sqrt{\hat{\mathcal{A}}\,(\mathrm{T}\Gamma_a)}}{\sqrt{\hat{\mathcal{A}}\,(\mathrm{N}\Gamma_a)}} \quad (6)$$

with $\mu_5 = \frac{1}{(2\pi)^5\alpha'^3}$. Here $\mathrm{T}\Gamma_a$ denotes the tangent bundle and $\mathrm{N}\Gamma_a$ the normal bundle of the D5-brane in X. The gauge group on such a stack of D5-branes is $SP(2N_a)$. If we also allowed for $2M$ D9-branes with trivial

gauge bundle, these would support an additional $SO(2M)$ gauge factor. For shortness we do not explicitly include these branes in our formulas, but this is easily accomplished.

From the CS terms it is straightforward to derive the tadpole cancellation condition for C_{10} and C_6

$$\begin{aligned} \sum_{i=1}^{K} N_i\, n_i &= 16, \\ \sum_{i=1}^{K} N_i\, \mathrm{ch}_2(V_i) - \sum_{a=1}^{L} N_a\, \gamma_a &= -c_2(T), \end{aligned} \tag{7}$$

where γ_a denotes the Poincare dual 4-form of the 2-cycle Γ_a.

3. MASSLESS SPECTRUM

The chiral massless spectrum resulting from open strings stretched between the different stacks of D9 and D5-branes is determined by the respective Euler characteristics

$$\chi(W) = \sum_{r=0}^{3} (-1)^r \dim H^r(X,W) = \int_X \left(\mathrm{ch}_3(W) + \frac{1}{12}\, c_1(W)\, c_2(T) \right) \tag{8}$$

listed in Table 1.

reps.	$\prod_{i=1}^K SU(N_i) \times U(1)_i \times \prod_{a=1}^L SP(2N_a)$
$(\mathbf{Sym}_{U(N_i)})_{2(i)}$	$\chi(\bigwedge^2 V_i)$
$(\mathbf{Anti}_{U(N_i)})_{2(i)}$	$\chi(\bigotimes_s^2 V_i)$
$(\mathbf{N}_i, \mathbf{N}_j)_{1(i),1(j)}$	$\chi(V_i \otimes V_j)$
$(\mathbf{N}_i, \overline{\mathbf{N}}_j)_{1(i),-1(j)}$	$\chi(V_i \otimes V_j^*)$
$(\mathbf{N}_i, 2\mathbf{N}_a)_{1(i)}$	$\chi(V_i \otimes \mathcal{O}\|_{\Gamma_a})$

Table 1: Chiral massless spectrum.

For the massless D5-brane matter we have described the D5-brane wrapping the 2-cycle Γ_a by the skyscraper sheaf $\mathcal{O}|_{\Gamma_a}$ supported on the 2-cycle

Γ_a. The Chern classes of this sheaf are $\mathrm{ch}(\mathcal{O}|_{\Gamma_a}) = (0, 0, -\gamma_a, 0)$ implying

$$\chi(V_i \otimes \mathcal{O}|_{\Gamma_a}) = -\int_{\Gamma_a} c_1(V_i). \tag{9}$$

The non-chiral massless spectrum can be determined from the respective cohomology groups $H^*(X, V \otimes W^*)$ or more generally, if non locally free sheaves are involved, from the extensions $\mathrm{Ext}_X(V, W)$. In addition, there exists non-chiral adjoint matter counted by $H^1(X, V_i \otimes V_i^*)$ for the D9-branes and anti-symmetric matter counted by $H^1(\Gamma_a, \mathcal{O})$ plus $H^0(\Gamma_a, \mathrm{N}\Gamma_a)$ for the D5-branes [16]. Here $\mathrm{N}\Gamma_a$ denotes the normal bundle of the 2-cycle Γ_a in X.

One can show that for the chiral matter in Table 1 the non-abelian gauge anomalies in four dimensions precisely cancel if the tadpole cancellation conditions (7) are satisfied.

4. K-THEORY CONSTRAINTS

It is well known that in intersecting D-brane models, besides the R-R tadpole cancellation condition additional torsion constraints arise due to the existence of stable non-BPS branes classified by K-theory [15]. From the effective field theory, these constraints guarantee the absence of $SP(2N)$ global Witten anomalies on probe branes carrying such symplectic gauge fields. In our case these are precisely the D5-branes wrapping 2-cycles of the Calabi-Yau X. Therefore, the cancellation of the Witten anomaly leads to the constraint

$$\sum_{i=1}^{K} N_i\, \chi(V_i \otimes \mathcal{O}|_{\Gamma_a}) = 0 \bmod 2 \tag{10}$$

for every 2-cycle Γ_a. Therefore, this condition is precisely the condition for the entire vector bundle $W = \bigoplus_{i=1}^{K} N_i\, V_i$ to admit spinors

$$c_1(W) = \sum_{i=1}^{K} N_i\, c_1(V_i) = 0 \bmod 2. \tag{11}$$

Note that for the heterotic string this condition was derived from the absence of anomalies in the two-dimensional non-linear sigma model [17, 18].

5. GREEN-SCHWARZ MECHANISM

Since all these string models naturally contain abelian gauge groups, one also has mixed abelian-non-abelian, mixed abelian-gravitational and cubic

abelian anomalies. As usual in string theory these anomalies do not cancel directly but only after axionic couplings are taken into account. Let us briefly summarize at least for the three mixed anomalies how the generalized Green-Schwarz mechanism works in this case. The mixed $U(1)_i - SU(N_j)^2$ anomaly for $i \neq j$ is given by

$$\begin{aligned} A_{i;jj} &= N_i \Big(\chi(V_i \otimes V_j) + \chi(V_i \otimes V_j^*)\Big) \\ &= 2N_i \int_X \left[n_j \mathrm{ch}_3(V_i) + c_1(V_i)\, \mathrm{ch}_2(V_j) + \frac{n_j}{12}\, c_1(V_i)\, c_2(T) \right] \end{aligned} \quad (12)$$

The last expression also holds for the case $i = j$, where also the contribution from the symmetric and antisymmetric matter and the tadpole constraint have to be taken into account. The mixed $U(1)_i - SP(N_a)^2$ anomaly is

$$A_{i;aa} = N_i\, \chi(V_i \otimes \mathcal{O}|_{\Gamma_a}) = -N_i \int_X c_1(V_i) \wedge \gamma_a. \quad (13)$$

For the mixed $U(1)_i - G^2$ anomaly one finds

$$\begin{aligned} A_{i;GG} &= \sum_{j \neq i} N_i\, N_j \Big(\chi(V_i \otimes V_j) + \chi(V_i \otimes V_j^*)\Big) + \sum_a 2N_i\, N_a\, \chi(\mathcal{O}|_{\Gamma_a} \otimes V_i^*) + \\ & \quad N_i\,(N_i - 1)\, \chi(\textstyle\bigotimes_s^2 V_i) + N_i\,(N_i + 1)\, \chi(\textstyle\bigwedge^2 V_i) \\ &= N_i \int_X \left[24\, \mathrm{ch}_3(V_i) + \frac{1}{2}\, c_1(V_i)\, c_2(T) \right]. \end{aligned} \quad (14)$$

These anomalies have to be canceled by axionic Green-Schwarz couplings arising from the dimensional reduction of the three kinds of CS-terms (1,3,6). We expand the relevant two and six-forms as

$$C_2 = C_0^{(2)} + \ell_s^2 \sum_{k=1}^{h_{11}} C_k^{(0)}\, \omega_k, \quad C_6 = \ell_s^6\, C_0^{(0)}\, \mathrm{vol}_6 + \ell_s^4 \sum_{k=1}^{h_{11}} C_k^{(2)}\, \widehat{\omega}_k, \quad (15)$$

where ω_k and $\widehat{\omega}_k$ are a normalized basis of 2- and 4-cycles on X with $\int \omega_k \wedge \widehat{\omega}_l = \delta_{kl}$. The four-dimensional 2-forms $C_0^{(2)}, C_k^{(2)}$ are Hodge dual to the four-dimensional scalars $C_0^{(0)}, C_k^{(0)}$.

By dimensional reduction we obtain the following axionic mass terms in four-dimensions

$$\begin{aligned} M_0 &= \frac{1}{6\,(2\pi)^5 \alpha'} \sum_i N_i \int_{\mathbb{R}^{1,3}} C_0^{(2)} \wedge f_i \int_X \left[\mathrm{Tr}_{n_i} \overline{F}_i^3 - \frac{1}{16}\, \mathrm{Tr}_{n_i} \overline{F}_i \wedge \mathrm{tr} \overline{R}^2 \right], \\ M_k &= \frac{1}{(2\pi)^2\, \alpha'} \sum_i N_i \int_{\mathbb{R}^{1,3}} C_k^{(2)} \wedge f_i \left[\mathrm{Tr}_{n_i} \overline{F}_i \right]_k, \end{aligned} \quad (16)$$

where f_i denotes the field strength of the $U(1)_i$ observable gauge group and $\overline{F}_i$ the field strength of the internal gauge field. The traces Tr_{n_i} are over the fundamental representation of the structure group $U(n_i)$. We have expanded

$$\mathrm{Tr}\overline{F}_i = (2\pi) \sum_k \left[\mathrm{Tr}\overline{F}_i\right]_k \, \omega_k. \tag{17}$$

Similarly one obtains the vertex couplings for $C_0^{(0)}$

$$V_0 = \frac{1}{2\,(2\pi)} \sum_i n_i \int_{\mathbb{R}^{1,3}} C_0^{(0)} \wedge \mathrm{Tr}_{N_i} F_i^2 - \frac{1}{4\,(2\pi)} \int_{\mathbb{R}^{1,3}} C_0^{(0)} \wedge \mathrm{tr} R^2 \tag{18}$$

and for $C_k^{(0)}$

$$\begin{aligned} V_k \;\; = \;\; & \frac{1}{4\,(2\pi)} \sum_i \int_{\mathbb{R}^{1,3}} C_k^{(0)} \wedge \mathrm{Tr}_{N_i} F_i^2 \; \left[\mathrm{Tr}_{n_i}\overline{F}_i^2 - \frac{n_i}{48} \mathrm{tr}\overline{R}^2\right]_k - \\ & \frac{1}{4\,(2\pi)} \sum_a \int_{\mathbb{R}^{1,3}} C_k^{(0)} \wedge \mathrm{Tr}_{SP(2N_a)} F_a^2 \; [\gamma_a]_k - \\ & \frac{1}{768\,(2\pi)} \int_{\mathbb{R}^{1,3}} C_k^{(0)} \wedge \mathrm{tr} R^2 \; \left[\mathrm{tr}\overline{R}^2\right]_k . \end{aligned} \tag{19}$$

Here we have expanded

$$\mathrm{Tr}\overline{F}_i^2 = (2\pi)^2 \sum_k \left[\mathrm{Tr}\overline{F}_i^2\right]_k \widehat{\omega}_k, \qquad \gamma_a = \sum_k [\gamma_a]_k \, \widehat{\omega}_k \tag{20}$$

and similarly for the internal curvature. Note that in the derivation of these vertex couplings also the D5-branes gave a contribution and that the tadpole cancellation conditions had to be used to bring the expression to its final form (19). Now we can combine the axionic mass and vertex couplings to provide counter terms for the triangle anomalies. Indeed, adding up all these graphs yields precisely an expression of the form of the mixed anomalies (12,13,14).

As usual the anomalous $U(1)$ gauge fields become massive via the Green-Schwarz couplings, where the longitudinal polarisations are given by some of the massive axionic fields.

6. FAYET-ILIOPOLOUS TERMS

From the general analysis of four-dimensional $\mathcal{N}=1$ supergravity it is well-known that the coefficients ξ_i of the FI-terms can be derived from the Kähler potential $\mathcal{K}$ via the relation

$$\frac{\xi_i}{g_i^2} = \left.\frac{\partial \mathcal{K}}{\partial V_i}\right|_{V=0}, \tag{21}$$

where the gauge invariant Kähler potential relevant for our type of construction reads

$$\mathcal{K} = \frac{M_{pl}^2}{8\pi}\Bigg[-\ln\Big(S+S^* - \sum_x Q_0^i\, V_i\Big) - \ln\Big(-\sum_{k,l,m=1}^{h_{11}} \frac{d_{klm}}{6}\Big(T_k+T_k^* - \sum_i Q_k^i\, V_i\Big)$$
$$\Big(T_l+T_l^* - \sum_i Q_l^i\, V_i\Big)\Big(T_m+T_m^* - \sum_i Q_m^i\, V_i\Big)\Big)\Bigg]. \qquad (22)$$

The charges Q_k^i are defined via

$$S_{mass} = \sum_{i=1}^{K}\sum_{k=0}^{h_{11}} \frac{Q_k^i}{2\pi\alpha'}\int_{\mathbb{R}_{1,3}} f_i \wedge C_k^{(2)} \qquad (23)$$

and can easily be extracted from the mass terms (16).

This results in the FI-terms

$$\frac{\xi_i}{g_i^2} \simeq \frac{1}{2}\int_X J\wedge J\wedge \mathrm{Tr}_{n_i}\overline{F}_i - \frac{(2\pi\alpha')^2}{3!}\int_X \left[\mathrm{Tr}_{n_i}\overline{F}_i^3 - \frac{1}{16}\,\mathrm{Tr}_{n_i}\overline{F}_i\wedge \mathrm{tr}\overline{R}^2\right]. \qquad (24)$$

Since they depend on the Kähler moduli, though exact in sigma model perturbation theory, one expects these expressions to be corrected by worldsheet instanton contributions. Supersymmetry implies that the D-terms have to vanish, which for zero VEVs for charged matter fields means that all FI-terms have to vanish. Note that setting (24) to zero is nothing else than the non-abelian generalization of the MMMS equation also including curvature terms.

The FI-term can be written as the imaginary part of a central charge

$$\frac{\xi_i}{g_i^2} \simeq \mathrm{Im}\left(\int_X \mathrm{Tr}_{n_i}\left[e^{-i\varphi}\, e^{2\pi\alpha' F - iJ}\,\sqrt{\hat{A}(X)}\right]\right) \qquad (25)$$

with $\varphi = \pi/2$. This is precisely the perturbative part of the expression appearing in the Π-stability condition of [12]. In the case of an $\Omega\sigma$ orientifold with O7- and induced O3-planes, for the introduced pairs of $D9-\overline{D9}$ branes one would get a similar result with $\varphi = 0$.

6. GAUGE KINETIC FUNCTIONS

Let us now give the expressions for the gauge kinetic functions. The holomorphic gauge kinetic function f_i appears in the four dimensional effective field theory as

$$\mathcal{L}_{YM} = \frac{1}{4}\,\mathrm{Re}(\mathrm{f}_i)\, F_i\wedge \star F_i + \frac{1}{4}\,\mathrm{Im}(\mathrm{f}_i)\, F_i\wedge F_i. \qquad (26)$$

With the definition of the complexified dilaton and Kähler moduli

$$S = \frac{1}{2\pi}\left[e^{-\phi_{10}}\frac{\mathrm{Vol}(\mathcal{M})}{\ell_s^6} + i\, C_0^{(0)}\right], \qquad T_k = \frac{1}{2\pi}\left[-e^{-\phi_{10}}\alpha_k + iC_k^{(0)}\right], \tag{27}$$

the gauge kinetic functions can be deduced from their imaginary parts in the vertex couplings (18,19)

$$\mathrm{f}_{SU(N_i)} \;=\; 2n_i\, S + \sum_{k=1}^{h_{11}} T_k \left[\mathrm{Tr}_{n_i}\overline{F}_i^2 - \frac{n_i}{48}\mathrm{tr}\overline{R}^2\right]_k. \tag{28}$$

The real part of the holomorphic gauge kinetic function f_i can be cast into the form

$$\mathrm{Re}(\mathrm{f}_i) = \frac{1}{\pi\ell_s^6 g_s}\left[\frac{n_i}{3!}\int_X J\wedge J\wedge J - \frac{(2\pi\alpha')^2}{2}\int_X J\wedge\left(\mathrm{Tr}_{n_i}\overline{F}_i^2 - \frac{n_i}{48}\mathrm{tr}\overline{R}^2\right)\right] \tag{29}$$

and further be written as

$$\mathrm{Re}(\mathrm{f}_i) \simeq \mathrm{Re}\left(\int_X \mathrm{Tr}_{n_i}\left[e^{-i\varphi}\, e^{2\pi\alpha' F - iJ}\sqrt{\hat{A}(X)}\right]\right) \tag{30}$$

with $\varphi = \pi/2$. For the D5-branes the gauge couplings are given by

$$\mathrm{Re}(\mathrm{f}_a) = \frac{1}{2\pi\ell_s^2 g_s}\int_{\Gamma_a} J. \tag{31}$$

7. SUPERSYMMETRY

For the mostly studied case of choosing just $U(1)$ bundles the supersymmetry condition was simply the vanishing of the FI-terms (24). In [11] arguments were presented that the non-integrated supersymmetry condition in the large radius limit reads

$$\left[\mathrm{Im}\left(e^{-i\varphi}\, e^{2\pi\alpha' F - iJ}\sqrt{\hat{A}(X)}\right)\right]_{\mathrm{top}} = 0. \tag{32}$$

The notion of stability relevant for (32) has been analysed in [19] and called π-stability (to stress that it is only the perturbative part of Π-stability [12]). In particular, the authors have shown that in the large radius limit (32) has a unique solution precisely if the bundle is stable with respect to the deformed slope

$$\pi(V) = -\mathrm{Arg}\left(\int_X \mathrm{Tr}_{n_i}\left[e^{2\pi\alpha' F - iJ}\sqrt{\hat{A}(X)}\right]\right), \tag{33}$$

i.e. the phase of the central charge. For supersymmetric configurations, we need to ensure that all objects are BPS with respect to the same supersymmetry algebra. This is guaranteed by the integrability condition given by the vanishing of the FI-terms (25) (with $\varphi = \pi/2$ in our case). For each stack of D9-branes one obtains one constraint on the Kähler moduli, so that a certain number of them, together with their axionic partners, are frozen (if we set all VEVs of charged fields to zero).

Not very much is known about π-stable bundles, but it has been shown that at large radius μ-stability implies π-stability [19], so that one can use the well studied class of μ-stable bundles for concrete model building.

8. TOWARDS STRING MODEL BUILDING

We have collected the main large radius model building ingredients and consistency conditions for the construction of chiral supersymmetric Type I models with D9-branes endowed with stable unitary bundles as well as D5-branes wrapping effective cycles. For the gauge kinetic functions and the FI-terms there will be further world-sheet instanton corrections, so that our analysis in only correct in the perturbative regime.

The next step is to do concrete model building and to look for Standard-like or GUT like models. So far this program has only been carried out for simple toroidal orbifold spaces with D9-branes endowed with just $U(1)$ bundles. Since one needs to have certain control over stable bundles, a good starting point is the spectral cover construction of μ-stable $SU(n)$ bundles on elliptically fibered Calabi-Yau spaces [20].

One way to define stable $U(n)$ bundles is via twisting an $SU(n)$ bundle with a line bundle on X. One starts with a stable bundle $SU(n)$ bundle V as it arises from the spectral cover construction of Friedman, Morgan and Witten [20, 21, 10]. In addition we take an arbitrary line bundle $\mathcal{Q}$ on X. Then one can define the twisted bundle $V_{\mathcal{Q}} = V \otimes \mathcal{Q}$, which has non-vanishing first Chern class unless $\mathcal{Q}$ is trivial. A bundle V is μ-stable if and only if $V \otimes \mathcal{Q}$ is stable for every line bundle $\mathcal{Q}$ [22, 23]. Therefore, all these twisted $U(n)$ bundles are μ-stable if the $SU(n)$ bundles are. A couple of semi-realistic models have been constructed in the S-dual heterotic framework in [24]. A statistical analysis similar to [25] would be an interesting task to perform.

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